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Design and optimisation of 3D-printed energy harvesters for railway bridges

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Abstract. This paper investigates energy harvesting on railway bridges. The type of harvester studied is a cantilever bimorph beam with a mass at the tip and a load resistance. These parameters are adjusted to find the optimal design that tunes the harvester to the fundamental frequency of the bridge. An analytical model based on a variational formulation to represent the electromechanical behaviour of the device is presented. The optimisation problem is solved using a genetic algorithm with constraints of geometry and structural integrity. Additive manufacturing to 3D print energy harvesters is used to maximise design flexibility and energy performance. The proposed procedure is implemented in the design and manufacture of an energy harvesting device for a railway bridge on an in-service high-speed line. The structure under study corresponds to a six-arched concrete bridge with double ballasted track. The proposed design is experimentally validated and the performance under operating conditions is evaluated in a test field.

1. Introduction

One of the major limiting factors in the implementation of sensor networks in railway monitoring applications is the lack of a long-term and low maintenance power supply. Most existing systems require battery changes, and difficulty access and infrequent maintenance operations can limit their practical implementation. In this sense, piezoelectric energy harvesting is becoming an alternative to the electrical supply of sensors and nodes in remote areas [1, 2]. Within the context of this work, many research projects have focused on the development of energy harvesters based on the piezoelectric effect to transform railway-induced vibrations into electrical energy to be used in small power devices and sensors. The most common typology is the bimorph cantilever beam. This system has the ability to generate energy from environmental vibrations in a frequency range of $3 - 100\,\mathrm{Hz}$ [3]. The predominant frequencies of vibrations caused by rail traffic in the infrastructure are within the above range, making it possible to supply low-power devices and sensors. An important problem of energy collection systems is associated with the fact that the performance of the energy production device is limited to a very narrow operating frequency band around its resonance frequency [4]. Moreover, the output power is drastically reduced when the excitation frequency deviates slightly from the resonance condition. This situation is due to the fact that the generator resonant frequency is often not tuned to the vibration frequency. Many researchers have focused on the development of resonance frequency tuning techniques for energy harvesters [5]. The main objective of this work is to estimate the optimal tuning frequency to which an energy harvester maximised its performance. A statistical process based on the analysis of the mechanical energy generated by a lumped-mass model from the bridge response due to train passage is presented. Moreover, the proposed tuning process is validated for a harvesting system in a railway bridge under operating conditions.

2. Tuning approach

The proposed approach considers the energy harvester attached to the bridge at a location defined by the coordinate x_b. This device is subjected to vertical vibration $z_b(x_b, t)$ induced by a train passage travelling at a speed V. Energy harvesting performance is closely related to the dynamic behaviour of the bridge during train passages [6]. Maximum performance would be obtained when the harvester is tuned to the fundamental mode shape of the bridge. Although the harvester device can be tuned to the natural frequency of the fundamental mode shape of the bridge, uncertainties in the dynamic response affect not only vibration levels but also frequency content, making the choice of the tuning frequency more difficult.

The dynamic response of a bridge under a train circulation is complex and is affected by several factors. The most obvious and certain are the bridge properties, the geometric scheme of the train axles, and the speed of circulation V. Furthermore, there are other factors that are much more uncertain in determining the response of the bridge [7], such as structural damping and various interaction mechanisms, the most relevant being vehicle-structure, track-structure, and soil-structure interaction effects. The dynamic response of a railway bridge due to train passage can be described by a stochastic process, assuming that the load has a random amplitude described by a Poisson process [8]. Moreover, Lombaert et al. [9] demonstrated that vibrations induced by railway traffic can be described by a complex random process that follows a chi-squared distribution with two degrees of freedom, which is a particular case of the gamma distribution $\Gamma(a, b)$ with $a = 1$ and $b = 1/2$.

The energy harvested by a frequency-tuned device depends on the bridge response given by the train type and speed, and the uncertainties involved in the above-mentioned interaction mechanisms. Therefore, due to the nature of the response caused by rail traffic, the energy collected from the bridge vibration can be described by a stochastic process. In this work, it is assumed that the energy harvested by a device tuned to the frequency ω follows a gamma distribution $\{E(\omega)\} \sim \Gamma(a, b)$ defined by the shape and rate parameters a and b, respectively. The mean value $\mu = a/b$, the standard deviation $\sigma = \sqrt{a/b^2}$ and the confidence interval $CI = \mu + 3\sigma$ of the energy harvested are derived from the cumulative distribution function (CDF).

The mechanical energy is used as an estimate of efficiency in the conversion of energy from the bridge vibration. A lumped-mass model is considered to represent the dynamic behaviour of the harvester. The properties of the system are given by the lumped mass m, the damping coefficient c and the stiffness k [6]. The damping coefficient c is used to represent the energy transfer from the bridge vibration to the harvester system [10]. The instantaneous power then corresponds to the power absorbed by the harvester plus the kinetic energy, and the mechanical energy in the system during the period T is a function of the velocity of the vibration:

$$E = c \int_0^T |\dot{y}(t)|^2 \, dt \tag{1}$$

where $y(t)$ denotes the vertical displacement of the lumped mass. The previous equation can be expressed in terms of the Fourier transform of the harvester velocity according to Parseval's theorem [6]. Then, the displacement of the harvesting device is expressed in terms of the vertical displacement of the bridge:

$$y(\overline{\omega}) = m\overline{\omega}^2 H(\beta) z_b(\mathbf{x}_b, \overline{\omega}) \tag{2}$$

with $H(\beta)$ the frequency response function of the harvester:

$$H(\beta) = \frac{1}{k}\left[\frac{1}{(1-\beta^2)+2\iota\zeta\beta}\right] \tag{3}$$

where $\beta = \overline{\omega}/\omega$ defines the relation between the load frequency $\overline{\omega}$ and the natural frequency ω of the lumped mass model, ζ is the damping coefficient and the imaginary unit number is denoted by the Greek letter ι. The natural frequency corresponds to the tuning frequency.

Thus, the energy collected becomes:

$$E = c\int_{-\infty}^{\infty} \left|\iota\overline{\omega}m\overline{\omega}^2 H(\beta) z_b(\mathbf{x}_b, \overline{\omega})\right|^2 d\overline{\omega} \tag{4}$$

Therefore, the tuning frequency is selected following a statistical procedure. The proposed procedure considers the study of the scattering of the mechanical energy of different harvesters tuned from 1 to 100 Hz to cover a wide range of frequencies, including most natural frequencies of railway bridges. The mechanical energy is computed for each train passage using experimental records of the bridge accelerations induced by train passages over the bridge under study. Then the maximum peak values of mechanical energy in a frequency range around the maximum of a confidence interval ($\Delta\omega$) are analysed. The frequency of these peaks is supposed to follow a Gaussian distribution $N(\widehat{\mu}, \widehat{\sigma})$ where $\widehat{\mu}$ is the mean and $\widehat{\sigma}$ is the standard deviation. The accumulated energy harvested in a device tuned to the mean frequency $\widehat{\mu}$ is maximised in the described frequency range for each train passage. Therefore, the optimal tuning frequency is $\omega_{\text{opt}} = \widehat{\mu}$.

An important point in the proposed tuning process is to ensure that the peak energy distribution follows a Gaussian distribution $N(\widehat{\mu}, \widehat{\sigma})$ in the frequency interval $\Delta\omega$. For this purpose, two conditions are proposed: *i*) the one-sample Kolmogorov-Smirnov test [11] should satisfy the null hypothesis that the peak energy frequency comes from a Gaussian distribution, and *ii*) the quantile-quantile representation fits the input sample with a determination coefficient $R^2 \geq 0.9$.

The tuning process is subdivided into three steps: *i*) experimental measurement of the bridge response to railway traffic; *ii*) computation of mechanical energy for different train passages for harvesters tuned in the frequency range of interest and *iii*) statistical characterisation of the energy distribution $\{E(\omega)\} \sim \Gamma(a, b)$ and estimation of the tuning frequency ω_{opt}.

3. Experimental tests

An experimental campaign was carried out to evaluate the energy harvesting performance on a railway bridge in July-September 2022. The bridge under study belongs to HSL Madrid-Sevilla. It is a ballasted double-track concrete arch bridge with six spans and total length of 25 m (see Figure 1). The tracks gauge is 1.435 m.

The tests were carried out in two stages. First, the bridge response was measured with a laser vibrometer in a preliminary test to estimate the tuning frequency. The experimental records allowed us to identify the optimal tuning frequency and the manufacturing of the harvester. Then, the proposed procedure was experimentally validated in a second stage with a comprehensive test consisting of measurement of bridge response at several points from ambient and forced vibrations. Also, the harvester performance was evaluated in this stage. The result of this test allowed us to characterise the dynamic properties of the structure along with the dynamic response of the bridge under railway traffic, the analysis of energy harvesting, and the validation of a harvester prototype in a relevant environment.

Figure 1. HSL bridge under study (38°13'41.06"N 4°32'40.58"W).

The experimental setup (Figure 2) consisted of an Ometron VH-1000-D laser vibrometer used in the first test to register the vibrations produced by rail traffic. The laser vibrometer was also used in the second test to measure the harvester response. The laser sensitivity was 8.0064 V/m/s. In the second test, piezoelectric accelerometers with nominal sensitivity of 10 V/g and lower frequency limit of approximately 0.1 Hz were used to measure the vibrations on the bridge. The Analog/Digital (A/D) conversion was performed with a LAN-XI portable acquisition system from Brüel & Kjaer. The A/D was performed at a high sampling frequency, which avoided aliasing effects. The recordings were decimated (to 256 Hz) to perform data analysis in the frequency range of interest. The response was filtered by applying a third-order Chebyshev filter with high-pass frequency of 1 Hz.

(a) (b)

Figure 2. Experimental setup: (a) circuit board, energy harvester and acceletrometer view (from left to right); and (b) vibrometer laser and laptop.

3.1. Tuning procedure

The response of the bridge under operating conditions was measured in July 2022 to estimate the harvester tuning frequency. This frequency was used to manufacture an energy harvester prototype. The measurement point was located in the middle of the intrados key point of arch number 4 (see Figure 1). A total of 21 train circulations were recorded between 14:40 and 20:00 hours.

Then, bridge response measurements were used in the estimation of the mechanical energy produced by harvesters tuned in the frequency range $1-100\,\text{Hz}$ with a damping ratio of $\zeta = 1\,\%$. Figure 3 shows the bridge response under three characteristic train passages for different energy level production. The train passages represented are Renfe S100 in duplex configuration at $V = 197.8\,\text{km/h}$ (passage #6), Renfe S100 travelling at $V = 218.7\,\text{km/h}$ (passage #10) and Renfe S120 at $V = 220.0\,\text{km/h}$ (passage #19). The mechanical energy was estimated using Equation (4). Then, the statistical procedure described in Section 2 was performed and the mechanical peak energies were analysed to obtain the optimal tuning frequency. Figure 4 shows the mechanical energy generated for the previous train passages. The peak energy frequency in Figure 4 was 42.27 Hz for passage #6, 43.10 Hz for passage #10 and 40.51 Hz for passage #19. The distribution of the peak energy presents dispersion in the frequency values.

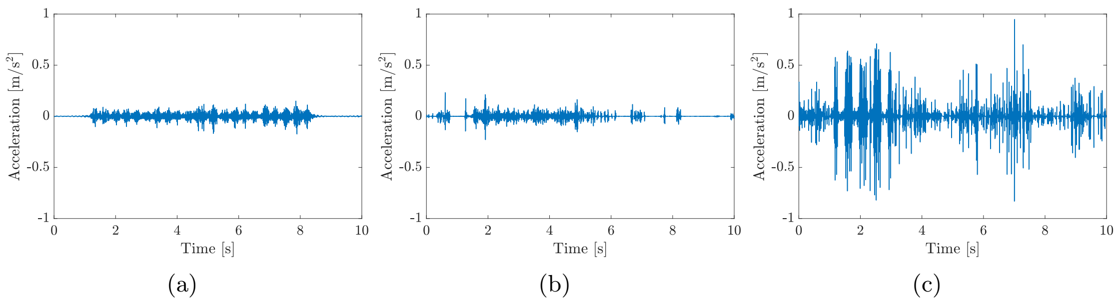

Figure 3. Time history of bridge acceleration for (a) passage #6, (b) passage #10 and (c) passage #19.

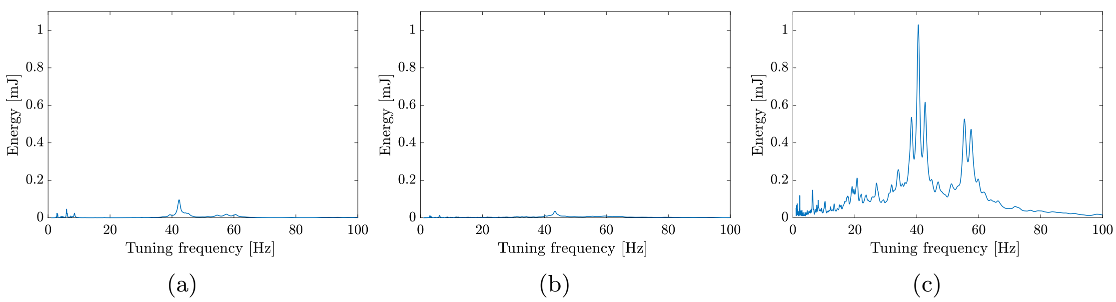

Figure 4. Mechanical energy harvested for (a) passage #6, (b) passage #10 and (c) passage #19.

Figure 5.(a) shows the statistical characterisation of the mechanical energy obtained from the total train circulations. The confidence interval of the energy distribution shows peak energies around low frequencies, which correspond to the bogie passage frequency and its harmonics. Furthermore, the energy distribution reached high levels in the frequency interval $\Delta\omega \in [30, 70]\,\text{Hz}$. However, the maximum values of the energy distribution were found around 40 Hz and 55 Hz. These values were indicative of a predominant frequency in the bridge response. The bogie passage frequency was not considered in the tuning procedure due to uncertainties related to the train configuration and travel speed. The maximum value of the confidence interval of the energy distribution was obtained at 39.6 Hz.

The tuning frequency was estimated from the maximum energy generated for each circulation in the interval $\Delta\omega \in [30, 70]$ Hz. The peak energy frequencies in this interval followed a Gaussian distribution given by $N(42.2, 5.1)$ (expressed in hertz). The one-sample Kolmogorov-Smirnov failed to reject the null hypothesis that the peak energy frequency comes from a Gaussian distribution. Furthermore, Figure 5.(b) compares the quantile-quantile plot of peak energy frequencies for each train passage (input sample) and the theoretical quantiles of the Gaussian distribution with mean $\widehat{\mu} = 42.2$ Hz and standard deviation $\widehat{\sigma} = 5.1$ Hz. The results were found to be in good agreement with a high determination coefficient $R^2 = 0.96$. Therefore, the optimal tuning frequency was estimated as the mean value of the Gaussian distribution.

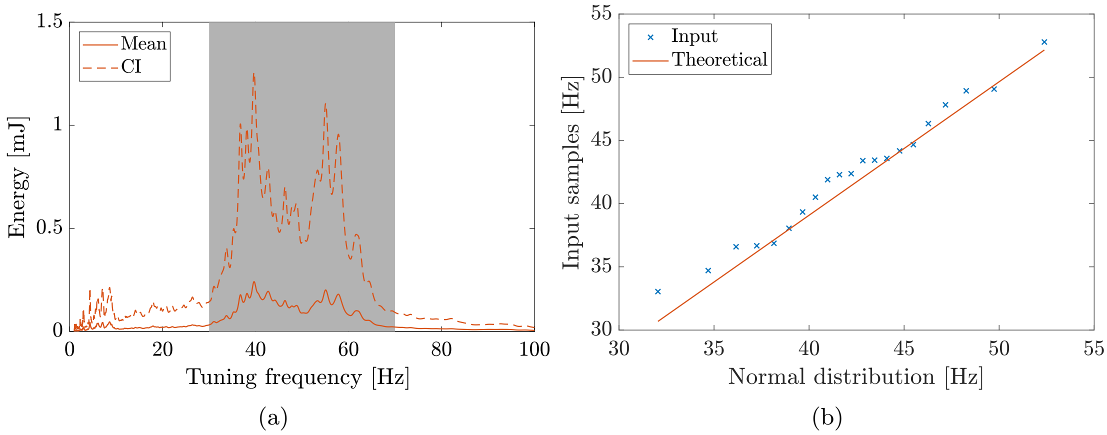

Figure 5. (a) Energy distribution and (b) quantile-quantile plots in the HSL bridge under study.

3.2. Manufacture

Once the tuning frequencies have been estimated, the energy harvester was manufactured following the optimal design procedure proposed in Reference [13]. The harvester configuration was a bimorph cantilever beam with a tip mass M_t and two perfectly bonded piezoelectric patches (PZT) to the substructure. The piezoelectrics were polarised in opposite directions along the thickness of the plate and connected in series. The system fed a load resistance R_l. The design parameters of the harvester were the thickness and length of the substructure, h_s and L_s, and the tip mass M_t.

The pizoelectric patch selected for this application was the commercial DuraAct patch transducer P-876.A12 [14], composed of a piezoelectric layer covered by copper electrodes. This patch is embedded in a structure mechanically pre-stressed by a polymer surface, making it flexible. The properties of the PZT material are available in [13].

The manufacturing method for the substructure was 3D printed fused deposition modelling (FDM). Among the variety of printing materials, a high temperature polyamide carbon fibre reinforced material (PAHT CF15) was used, as in some applications, to replace metals due to its high strength. The properties of the PAHT CF15 material were experimentally estimated according to the ASTM D638-14 standard [15]. The obtained material properties were: mean Young's modulus $E_s = 7.40$ GPa with a standard deviation of $s_E = 0.33$ GPa, and tensile strength $\sigma_y = 94$ MPa with a standard deviation of $s_{\sigma_y} = 9.9$ MPa.

The tuning procedure optimised the power dissipated by the load resistance R_l under resonant conditions. Then, the harvester design was obtained from the solution of an optimisation problem with constraints. Thus, the optimal design parameters for a harvester tuned to 42.2 Hz were $h_s = 0.96\,\mathrm{mm}$, $L_s = 74\,\mathrm{mm}$, $M_t = 26.3\,\mathrm{kg}$ and $R_l = 104.1\,\mathrm{k\Omega}$. Figure 6 shows the printed prototype with the piezoelectric patches glued to the substructure.

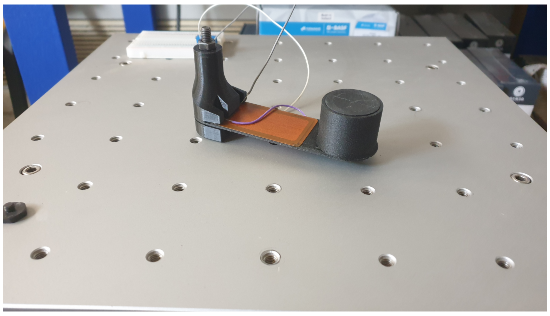

Figure 6. Energy harvester prototype.

3.3. Design verification

The performance of the prototype was verified in the laboratory by comparing the experimental and analytical frequency response functions for the tip response and the output voltage [13]. The experimental test consisted of measuring the response of the device due to the base excitation induced by an APS 400 ELECTRO-SEIS electrodynamic shaker. The tip velocity was measured by the laser vibrometer, and the output voltage was recorded directly with the acquisition system. Furthermore, the base acceleration was measured with an accelerometer with nominal sensitivity 100 mV/g. The device was subjected to a burst random acceleration in a 50 Hz bandwidth during 128 s. The burst percentage was established at 50 % of the total test time.

Figure 7 shows the experimental FRF and the analytical solution [13] for tip acceleration and voltage. The agreement between both sets is good. The damping coefficient ζ and the electromechanical coupling factor α were investigated by fitting the analytical solution of the FRF to the experimental response. The Levenberg-Marquardt non-linear least squares algorithm [16] was used to fit the frequency response function for tip acceleration and voltage. The analytical and experimental values for both parameters were $\zeta_{Ana} = 1.2\%$ and $\alpha_{Ana} = 1.1 \times 10^{-3}\,\mathrm{N/V}$, and $\zeta_{Exp} = 1.27\%$ and $\alpha_{Exp} = 0.97 \times 10^{-3}\,\mathrm{N/V}$, respectively. The agreement in the results verified the design of the energy harvesters.

4. Assessment of energy harvester performance in operating conditions

Finally, this section presents the energy harvested under operating conditions. On 8 September 2022, 16 passenger train circulations were recorded between 16:19 and 19:13 hours. All trains were RENFE High-Speed services. The energy harvester was located in the middle of the intrados key point in arch number 4 (see Figure 1. Table 1 summarises the train circulation, travel speed, and energy harvested. The energy levels varied in a wide range depending on the train type and the travelling speed. The results showed train passages of maximum energy, such as #10 and passages near-zero energy levels, such as #13. The total energy harvested was $E = 0.206\,\mathrm{mJ}$.

Figure 8 shows the time history and frequency content of the bridge acceleration and output voltage induced by a Renfe S100 travelling at $V = 210.6\,\mathrm{hm/h}$ on track 1 (passage #10). The frequency content of the bridge acceleration was found mainly around the tuning frequency. The harvester response was highly amplified as is observed in the acceleration time history. Figure 9 shows that both the harvester performance due to a Renfe S100 circulating at $V = 207.9\,\mathrm{km/h}$ on track 2 (passage #13), was much lower than in the previous case. Furthermore, Figures 8

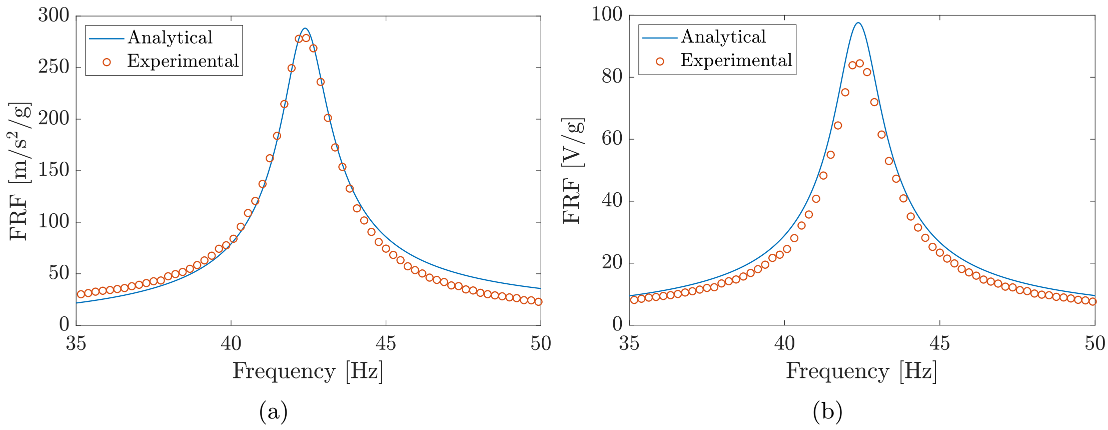

Figure 7. Analytical and experimental frequency response function of (a) tip acceleration and (b) voltage of a energy harvester tuned to 42.2 Hz.

Table 1. Energy harvesting in train passages recorded at the bridge under study.

Passage	Train	Track	V [km/h]	E [mJ]
1	S100	2	183.5	0.003
2	S102-duplex	2	206.2	0.009
3	S100	1	207.9	0.026
4	S100	2	205.2	0.003
5	S103	1	220.9	0.006
6	S130	1	202.5	0.019
7	S102-duplex	2	206.2	0.010
8	S100	2	197.1	0.004
9	S102	1	208.1	0.013
10	S100	1	210.6	0.046
11	S102	2	222.7	0.005
12	S100	1	207.9	0.018
13	S100	2	207.9	0.002
14	S100	2	199.8	0.027
15	S103	1	222.7	0.006
16	252	2	198.3	0.010
			Total	0.206

and 9 also include a comparison of the harvester response and the output voltage estimated from the bridge acceleration and the FRF obtained in the previous section. The agreement is good enough to validate the analytical solution [13] and allows the energy estimation in further design stages.

5. Conclusions

This research proposes and validates a tuning strategy for energy harvesters on railway bridges. The proposed procedure is based on the statistical distribution of mechanical energy generated by a lumped-mass model subjected to bridge vibrations induced by train passages. We have found that the mechanical energy produced by a harvester tuned over a frequency range presented

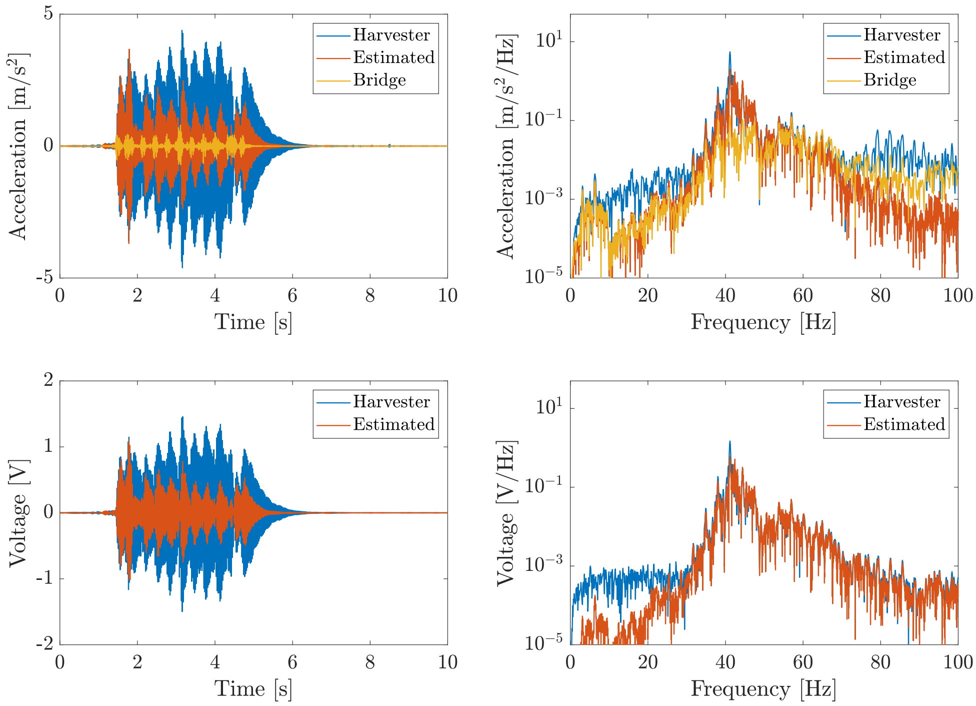

Figure 8. Bridge and harvester acceleration and voltage induced by Renfe S100 train circulating on track 1 at $V = 210.6\,\mathrm{km/h}$ (passage#10). The estimated results are also represented.

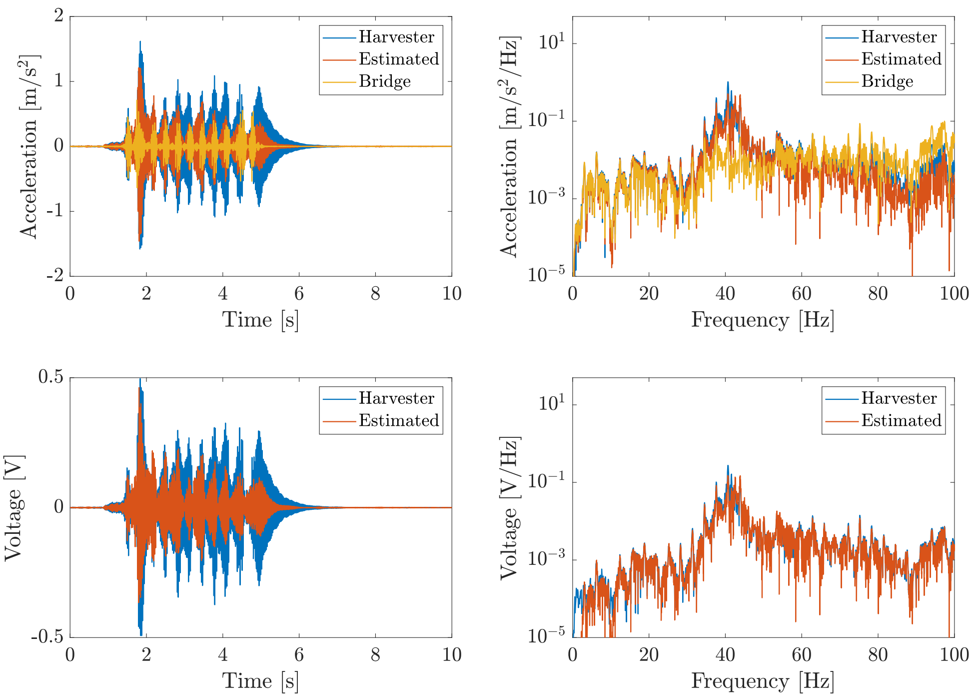

Figure 9. Bridge and harvester acceleration and voltage induced by Renfe S100 train circulating on track 2 at $V = 207.9\,\mathrm{km/h}$ (passage#13). The estimated results are also represented.

peak values that depended on the train configuration and speed. The energy levels followed a Gamma distribution, and the peak energy frequencies a Gaussian distribution. The mean value of the peak energy frequencies corresponded to the tuning frequency that maximised the harvester performance.

The proposed procedure has been experimentally validated on a railway bridge in an service high-speed line. The experimental tests consisted of measurement of bridge vibrations due to railway traffic, identification of the optimal tuning frequency, harvester manufacturing and the laboratory test. Subsequently, the performance of the device was evaluated under real operating conditions. The energy level depended on the train configuration and the travelling speed. The total energy harvested in a time window of three hours (16 train passages) was 0.206 mJ.

6. Acknowledgements
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